Recurrence relations and sequences.

1. Write down recurrence relations for each of the following sequences.

a) 10, 13, 16, 19, 22, . . . b) 2, 9, 16, 23, 30, . . .
c) 8, 9, 10, 11, 12, . . . d) 2, 4, 6, 8, 10, . . .
e) 5, $6\frac{1}{2}$, 8, $9\frac{1}{2}$, 11, . . . f) 7, 4, 1, -2, -5, . . .
g) 90, 80, 70, 60, 50, . . . h) 8, 16, 24, 32, 40, . . .
i) 5, 10, 20, 40, 80, . . . j) 1000, 200, 40, 8, 1.6, . . .
k) 25, 20, 16, 12.8, . . . l) 8, 12, 18, 27, 40 1/2, . . .

In each example write down a formula for u_n in terms of n.

2. Write down the first five terms in each of the following sequences

a) $u_{n+1} = u_n - 3$, $u_1 = 3$
b) $u_{n+1} = u_n - 1$, $u_1 = 7$
c) $u_{n+1} = 4u_n$, $u_1 = 1/2$
d) $u_{n+1} = 1/2\ u_n$, $u_1 = 2000$
e) $u_{n+1} = 2u_n - 3$, $u_1 = 4$
f) $u_{n+1} = 3u_n - 1$, $u_1 = 1$
g) $u_{n+1} = {}^{3}/_{4}\, u_n$, $u_1 = 256$

3. In each example find the first few terms and decide if a limit exists.
If it does, find it.
a) $u_{n+1} = 1/2\ u_n + 4$, $u_1 = 6$
b) $u_{n+1} = ¼\ u_n + 24$, $u_1 = 16$
c) $u_{n+1} = 2u_n + 1$, $u_1 = 4$
d) $u_{n+1} = 0.6u_n + 20$, $u_1 = 10$
e) $u_{n+1} = 0.9u_n + 3$, $u_1 = 40$

4. $u_n = 1/2n(n + 2)$ Find u_2 and u_7 and find n if $u_n = 24$.

5. $u_1 = -1$, $u_2 = 3$, $u_3 = 7$ Write down a recurrence relation for the sequence and find a formula for u_n .

6. $u_1 = 2$, $u_2 = 6$, $u_3 = 18$ Write down a recurrence relation for the sequence and find a formula for u_n .

7. If $S_n = 1/4n(n + 1)$, find u_1, u_2 and u_3 . Find a recurrence relation for the sequence and a formula for the u_n .

8. $u_n = 1/2\ n[1 - (-1)^n]$ Find u_{15} and u_{16}
List the first six terms of the sequence.

9. S_n is the sum of the first n terms of the sequence 1 , 0.5 , 0.25 , 0.125 , - - - -
Find S_6 , S_7 and S_8. Find the limit as $n \rightarrow \infty$

10. A sequence is defined by $u_n = 4n + 7$. Write down u_1, u_2, and u_3.
Evaluate $u_{n+1} - u_n$.

11. A sequence is defined by $u_{n+1} = 0.2u_n + 4$, $u_1 = 3$. Determine the limit of the sequence.

12. If $S_n = n^2 + 4n$, evaluate S_1, S_2 and S_3 and hence find the first three terms of the sequence.

13. Find the first three terms of the sequence in which sum of the first n terms is
$S_n = 3n^2 + 9n$.
Find a formula for u_n .

14. Find the fourth term of the sequence where the sum of the first n terms is
$S_n = n^3 - 2n$.

15. Find the tenth term of the sequence where the sum of the first n terms is
$S_n = 2^n + 1$.

16. A sequence is given by $u_{n+1} = 2u_n + 1$

Find expressions for u_1, u_2, and u_3 showing that $u_3 = 2^3u_0 + 2^2 + 2 + 1$

Deduce a similar expression for u_n .

17. A sequence is given by $u_{n+1} = 5u_n + 1$

Find expressions for u_1, u_2, and u_3 showing that $u_3 = 5^3u_0 + 5^2 + 5 + 1$

Deduce a similar expression for u_n .

18. A sequence is defined by $u_{n+1} = au_n + b$.

If $u_1 = 5$, $u_2 = 7$ and $u_3 = 13$ find a and b .

19. A sequence is given by $u_{n+1} = 5u_n + b$ where b is a constant.

If $u_1 = 8$ and $u_3 = 224$, find b .

20. Find the limit of the sequence $u_{n+1} = {}^3/_4\, u_n + 5$ with $u_1 = 12$.

21. Find the limit of the sequence $u_{n+1} = 0.8u_n + 14$

22. If $u_n = 2^n + 3$, list the first 5 terms. Find a and b such that $u_{n+1} = au_n + b$.

Recurrence relations and sequences. Solutions

	Recurrence relation	Formula
1. a)	$u_{n+1} = u_n + 3$, $u_1 = 10$	$u_n = 3n + 7$
b)	$u_{n+1} = u_n + 7$, $u_1 = 2$	$u_n = 7n - 5$
c)	$u_{n+1} = u_n + 1$, $u_1 = 8$	$u_n = n + 7$
d)	$u_{n+1} = u_n + 2$, $u_1 = 2$	$u_n = 2n$
e)	$u_{n+1} = u_n + 1.5$, $u_1 = 5$	$u_n = {}^3/_2\, n + {}^7/_2$
f)	$u_{n+1} = u_n - 3$, $u_1 = 7$	$u_n = 10 - 3n$
g)	$u_{n+1} = u_n - 10$, $u_1 = 90$	$u_n = 100 - 10n$
h)	$u_{n+1} = u_n + 8$, $u_1 = 10$	$u_n = 8n$
i)*	$u_{n+1} = 2u_n$, $u_1 = 5$	$u_n = 5 \,.\, 2^{n-1}$
j)*	$u_{n+1} = {}^1/_5\, u_n$, $u_1 = 1000$	$u_n = (1/5^{n-1}) \,.\, 1000$
k)*	$u_{n+1} = {}^4/_5\, u_n$, $u_1 = 25$	$u_n = 25 \,.\, 0.8^n$
l)*	$u_{n+1} = 1.5u_n$, $u_1 = 8$	$u_n = 8 \,.\, 1.5^{n-1}$

*{Note : * means not easy!}*

2. a) 8, 11, 14, 17, 10 b) 7, 6, 5, 4, 3
c) 1/2, 2, 8, 32, 128 d) 2000, 1000, 500, 250, 125
e) 4, 5, 7, 11, 19, 35 f) 1, 2, 5, 14, 41
g) 256, 192, 144, 108, 81
3. a) limit = 8 b) 32 c) no limit d) 50 e) 30
4. $u_2 = 4$, $u_7 = 31.5$ $1/2\, n(n + 2) = 24 \Rightarrow n = 6$ {or -8}
5. $u_{n+1} = u_n + 4$, $u_1 = -1$; $u_n = 4n - 5$
6. $u_1 = 2$, $u_2 = 6 = 2 \,.\, 3$, $u_3 = 18 = 2 \,.\, 3^2$,
$u_4 = 54 = 2 \,.\, 3^3$, $u_5 = 162 = 2 \,.\, 3^4$
$u_{n+1} = 3u_n$, $u_1 = 2$; $u_n = 2 \,.\, 3^n$
7. $S_1 = 1/2$, $S_2 = {}^3/_2$, $S_3 = 3$, $S_4 = 5$
$u_1 = 1/2$, $u_2 = S_2 - S_1 = 1$, $u_3 = S_3 - S_2 = 11/2$, $u_4 = S_4 - S_3 = 2$
$u_{n+1} = u_n + 0.5$, $u_1 = 0.5$; $u_n = 1/2\, n$

8. 1, 0, 3, 0, 5, 0 $u_{15} = 15$, $u_{16} = 0$ $u_n = n$ if n is odd
$u_n = 0$ if n is even

9. $S_1 = 1, S_2 = 1.5, S_3 = 1.75, S_4 = 1.875, S_5 = 1.9375,$

$S_6 = 1.96875, S_7 = 1.984375, S_8 = 1.992 \quad \lim S_n = 2$

10. $u_n = 4n + 7 \quad u_1 = 11, u_2 = 15, u_3 = 19 \quad u_{n+1} - u_n = 4$

11. Let L be the limit => $L = 0.2L + 4 \Rightarrow 0.8L = 4 \Rightarrow L = 4 / 0.8 = 5$

12. $S_1 = 5, S_2 = 12, S_3 = 21$; $u_1 = 5, u_2 = 7, u_3 = 9, u_4 = 11$; $u_n = 2n + 3$

13. $S_1 = 12, S_2 = 30, S_3 = 54$; $u_1 = 12, u_2 = 18, u_3 = 24$; $u_n = 6n + 6$

14. $S_3 = 21, S_4 = 56, U_4 = S_4 - S_3 = 56 - 21 = 35$

15. $S_9 = 513, S_{10} = 1025 \; u_{10} = S_{10} - S_9 = 1025 - 513 = 512$

16. $u_{n+1} = 2u_n + 1, u_1 = 2u_0 + 1,$

$u_2 = 2u_1 + 1 = 2[2u_0 + 1] + 1 = 2^2.u_0 + 2 + 1$

$u_3 = 2u_2 + 1 = 2[2^2.u_0 + 2 + 1] + 1 = 2^3u_0 + 2^2 + 2 + 1$

Similarly, $u_4 = 2^4.u_0 + 2^3 + 2^2 + 2 + 1$

$u_n = 2^n.u_0 + 2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots + 2 + 1$

17. $u_{n+1} = 5u_n + 1, u_1 = 5u_0 + 1,$

$u_2 = 5u_1 + 1 = 5[5u_0 + 1] + 1 = 5^2.u_0 + 5 + 1$

$u_3 = 5u_2 + 1 = 5[5^2.u_0+5+1]+1 = 5^3u_0 + 5^2 + 5 + 1$

Similarly, $u_4 = 5^4.u_0 + 5^3 + 5^2 + 5 + 1$

$u_n = 5^n.u_0 + 5^{n-1} + 5^{n-2} + 5^{n-3} + \ldots + 5 + 1$

18. $u_{n+1} = a.u_n + b \quad u_2 = a.u_1 + b \; 7 = 5a + b \ldots(1)$

$u_3 = a.u_2 + b \; 13 = 7a + b \ldots(2)$

=> $a = 3, b = -8$

19. $u_{n+1} = 5u_n + b \; u_2 = 5u_1 + b \; u_3 = 5u_2 + b = 5[5u_1 + b] + b = 25u_1 + 6b$

$224 = 25 \,.\, 8 + 6b \Rightarrow 6b = 24 \Rightarrow b = 4$

20. Let L be the limit $L = \frac{3}{4} L + 5 \Rightarrow L = 20$

21. $L = 0.8L + 14 \Rightarrow 0.2L = 14 \Rightarrow L = 70$

22. 5, 7, 11, 19, 35

Let $u_{n+1} = a.u_n + b$ and use with n = 1, 2

$5a + b = 7$ & $7a + b = 11 \Rightarrow a = 2, b = -3$

=> $u_{n+1} = 2u_n - 3, u_1 = 5$